New Simple Procedure for the Computation of the Multimode Admittance Matrix of Arbitrary Waveguide Junctions

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Abstract

The description of the microwave properties of waveguide junctions is a subject that has been studied considerably in the past. Of particular interest, form the engineering point of view, have been equivalent network representations (single mode) because of their very good computational efficiency. However, their efficiency was obtained taking into account only dominant mode interactions and such descriptions are no longer suitable for the design of modern microwave components. Higher order mode interactions have been subsequently accounted for using mode-matching procedures, however, the resulting codes, although accurate, can be computationally very heavy. It would therefore be desirable to develop equivalent network representations that could at the same time include higher-order interactions and lead to computationally efficient codes.

In this paper a simple method is described for the evaluation of the multimode network representation in terms of admittance parameters. The key feature of the method is that it starts from the wanted final results, the equivalent network representation, in order to obtain an analytic expression for the evaluation of the admittance matrix elements. The procedure is based on general network theory and is equivalent to ideally *measuring directly* the value of the admittance elements. In this paper the evaluation procedure is fully described. Measurement results of actual hardware are then compared with simulations indicating that the codes developed are indeed very accurate as well as computationally very efficient.

I - Introduction

The description of the microwave properties of waveguide junctions, like the one shown in Fig. 1, is a subject that has been studied considerably in the past. Of particular interest, from the engineering point of view, have been equivalent network representations, such as the ones that can be found in [1], for instance. The results in [1], however, only account for single mode interactions and are therefore not suitable for the design of modern microwave components. Higher order mode interactions have been traditionally accounted for using mode-matching procedures [2], however, the resulting codes, although accurate, can be computationally very inefficient. It would therefore be very desirable to develop equivalent network representations that could at the same time include higher-order interactions and lead to computationally efficient codes.

Recently, a multimode equivalent network representation for arbitrary waveguide junctions has been developed which is based on an integral equation formulation [3]. In [3], the imposition of the boundary conditions leads to an integral equation and to an equivalent network representation of the junction. Solving the integral equation for the unknown function allows for the subsequent computation of the multimode coupling matrix elements. Although this approach can be made computationally efficient, it is rather involved analytically so that a better approach was found necessary.

In this paper a simple method is described for the evaluation of the multimode network representation in terms of admittance parameters. The key feature of the method is that it starts from the wanted final results, the equivalent network representation, in order to obtain an analytic expression for the evaluation of the admittance matrix elements. The procedure is based on general network theory and is equivalent to ideally *measuring directly* the value of the admittance elements and was originally proposed for the study of waveguide T-junctions [4].

In this paper the procedure is fully described leading to simple and elegant analytical expressions that can be used for arbitrary waveguides. Admittance matrix descriptions for this type of problems have already been described in the past (see [5], for instance) but never obtaining such simple and elegant final expressions.

Measurement results of actual hardware are then compared with simulations indicating that the codes developed are indeed very accurate as well as computationally very efficient.

II - Theory

The structure under investigation is the junction between arbitrary waveguides shown in Fig. 1. The first step of the procedure is to define input and output reference planes (see Fig. 1), and to write formally the wanted final result in the form

$$I_m^{(\delta)} = \sum_{n=1}^{\infty} Y_{m,n}^{(\delta,1)} \cdot V_n^{(1)} + \sum_{n=1}^{\infty} Y_{m,n}^{(\delta,2)} \cdot V_n^{(2)} \qquad (1)$$

Where δ can be 1 for region (1) or 2 for region (2), as shown in Fig. 1. The above equation is equivalent in practice to the network representation shown in Fig. 2 and can be used to build computer codes to study structures involving single or cascaded junctions. The key point of the approach described in this paper is to note that equation (1) can also be used to obtain an analytical expression for the $Y_{m,n}$ elements. We can, in fact, write directly

$$Y_{m,n}^{(\delta,\gamma)} = \frac{I_m^{(\delta)}}{V_n^{(\gamma)}} \mid V_i^{(\xi)} = 0, i \neq n, \xi \neq \gamma. \qquad (2)$$

Where γ and ξ can be again 1 for region (1) or 2 for region (2). The above expression is the definition of $Y_{m,n}$ in terms of network theory and can also be used to *evaluate directly* the actual $Y_{m,n}$ elements. To explain how this can be done , let us begin with $Y_{m,n}^{(1,1)}$. For this elements, the definition in (2) requires a single mode incident from the left (port 1) (which corresponds

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to $V_n^{(1)} \neq 0$ and all other voltages in port 1 equal zero, thus $V_i^{(1)} = 0, i \neq n$) and a short circuit in port 2 (corresponding to all voltages in region (2) equal zero, thus $V_i^{(2)} = 0, \forall i$). The resulting structure becomes the one shown in Fig. 3 and, as we can see, no discontinuities are actually present so that we can write directly.

$$Y_{m,n}^{(1,1)} = (-j) \cdot Y_{0m}^{(1)} \cdot \cot(\beta_m^{(1)} l_{ref}) \cdot \delta_{m,n} \tag{3}$$

Following the same philosophy for the elements $Y_{m,n}^{(2,1)}$, the definition in (2) requires again a single mode incident from the left (port 1) and a short circuit in port 2. The current response will now be measured at port 2 using standard orthogonality and the same structure in Fig. 3. The resulting expression is

$$Y_{m,n}^{(2,1)} = (-j) \cdot \frac{Y_{0m}^{(1)}}{\sin(\beta_m^{(1)} l_{ref})} \cdot < \mathbf{e}_m^{(1)} \mathbf{e}_n^{(2)} > \tag{4}$$

which can also be used for $Y_{n,m}^{(1,2)}$ since the junction is lossless and reciprocal.

Finally, for the elements $Y_{m,n}^{(2,2)}$, the definition in (2) requires a single mode exciting from the right (port 2) and a short circuit at port 1. The situation is therefore as indicated in Fig. 4, the current response is again taken in port 2. Applying simple transmission line theory, the expression for the admittance elements can be written as:

$$Y_{m,n}^{(2,2)} = \sum_{k=1}^{\infty} (-j) Y_{0k}^{(1)} \cdot \cot(\beta_k^{(1)} l_{ref}) \cdot < \mathbf{e}_k^{(1)} \mathbf{e}_n^{(2)} > \cdot < \mathbf{h}_k^{(1)} \mathbf{h}_m^{(2)} > \tag{5}$$

It is interesting to observe that in this last equation the summation is extended over the modes of one waveguide only, namely region (1).

In all of the equations derived the symbol $<>$ stands for the scalar product of the quantities involved (resulting from the use of the orthogonality conditions) and **e** and **h** are the vector mode functions of the waveguides in the appropriate regions. $\beta_m^{(1)}$ is the propagation constant of the modes in region (1) and $Y_{0m}^{(1)}$ is the corresponding characteristic admittance. Moreover l_{ref} is the separation between the reference planes, as indicated in Fig. 1. Good convergence of the admittance elements has been observed taking such length of the order of a few millimeters. The step by step derivation of all the above equations will be discussed during the talk.

III - Application Examples

The present method was used to develop a software for the analysis of rectangular to rectangular and circular to rectangular waveguide junctions. Since analytical expressions are available for the modes in all regions [1], the scalar products can be easily computed and the admittance elements are then obtained by direct application of the above equations. For the circular to rectangular junction, however, the resulting coupling integrals have been evaluated numerically. The theory proposed in [6] and [7] is used in this case to convert the surface integrals to line integrals over the contour of the rectangular waveguide, thus increasing the speed of the computation.

Once the admittance coupling matrices are computed following this method, a complex structure, formed by cascading any discontinuity can be easily studied by building a global multimode equivalent network representation. An important feature of this method is that the analysis of the global network thus obtained requires only one inversion per frequency point of a system of linear equations which is banded, and therefore can be performed very efficiently. No intermediate inversions are required for the computation of the s-parameters of the complete structure, thus leading to codes that can be very fast even for very complex structures.

The software developed was first used to analyze a low-pass filter composed of 85 discontinuities including two double plane step transformers at the input and the output of the filter. The filter was designed by COM DEV Canada and manufactured by COM DEV U.K. The simulated results obtained with our software are shown in Fig. 5 and can be compared with the measured response of the hardware in fig. 6. It can be seen that the agreement is very good. The only factor of disagreement is in a frequency shift of about 200 MHz due to the fact that the hardware was manufactured with rounded corners while in the simulation all corners are assumed to be without any radius.

For this analysis 20 modes were used in the final equivalent network and 150 modes were used for the summation of the admittance elements. The analysis was performed on an IBM RISC 6000 platform and the time spent by the software for the analysis of the complete structure was 1 minute per each frequency point. The same analysis performed by COM DEV Canada using a proprietary software based on standard mode-matching takes about 30 minutes per frequency point.

As a further example, the analysis of a circular waveguide resonator with different input and output rectangular-window couplings was performed. In Fig. 7 we present the comparison between simulated results and measurements (the stars) showing a very good agreement. For this analysis 4 modes were used in all uniform waveguide sections and 120 modes were used to sum the admittance elements. The time spent by the software for the analysis of the complete structure was 2 minutes and 10 seconds for 100 frequency points.

IV - Conclusions

A simple and elegant procedure has been developed for the evaluation of the multimode equivalent network representation of the junction between arbitrary waveguides in terms of an admittance matrix. The expressions derived can be used for arbitrary waveguides and are particularly simple to use if the modes of at least one of the two waveguides involved are known analytically.

Comparison between the results of simulations obtained with software based on the procedure described and measurements of actual hardware have also been presented indicating that the new procedure indeed leads to very accurate and fast CAD tools.

Acknowledgement

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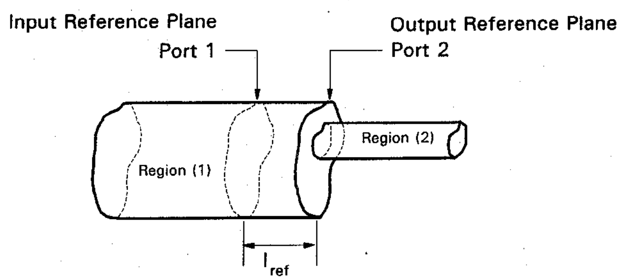


Fig. 1 Arbitrary waveguide junction studied in this paper

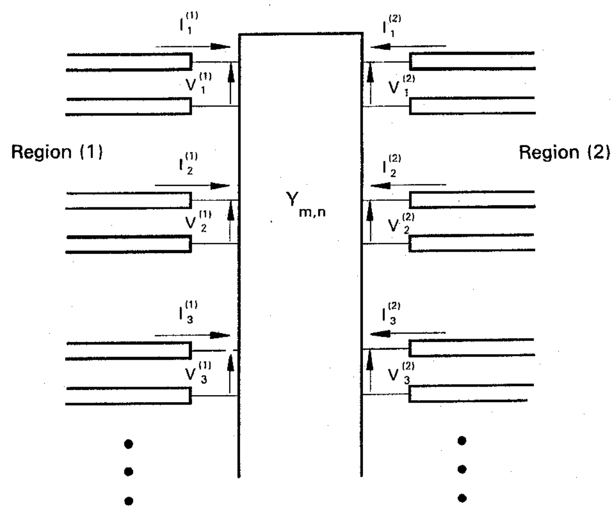


Fig. 2 Multimode equivalent network representation of the junction shown in Fig. 1, in terms of admittance parameters.

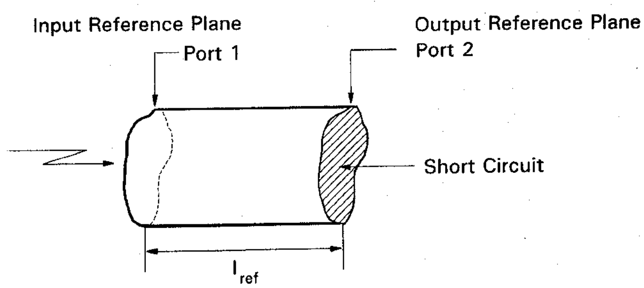


Fig. 3 Resulting structure used for the evaluation of the $Y_{m,n}^{(1,1)}$ and $Y_{m,n}^{(2,1)}$. A short circuit is placed at port 2.

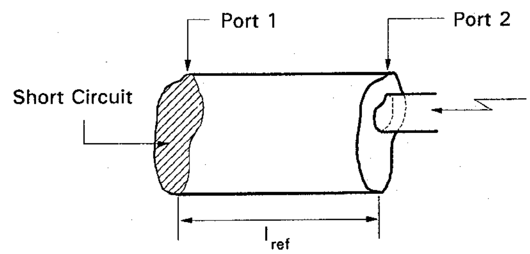


Fig. 4 Resulting structure used for the evaluation of the $Y_{m,n}^{(2,2)}$. A short circuit is placed at port 1.

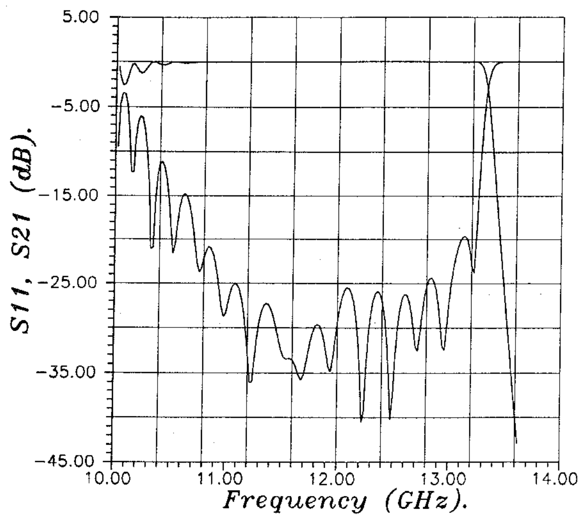


Fig. 5 Simulated results, obtained with the software developed, for the low-pass filter designed by COM DEV Canada. The filter is composed of 85 discontinuities.

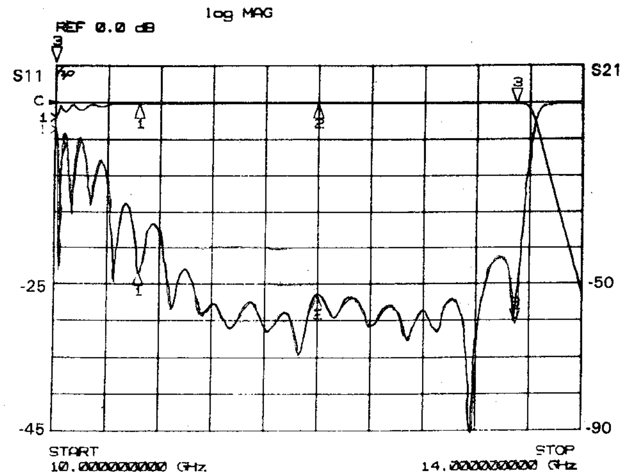
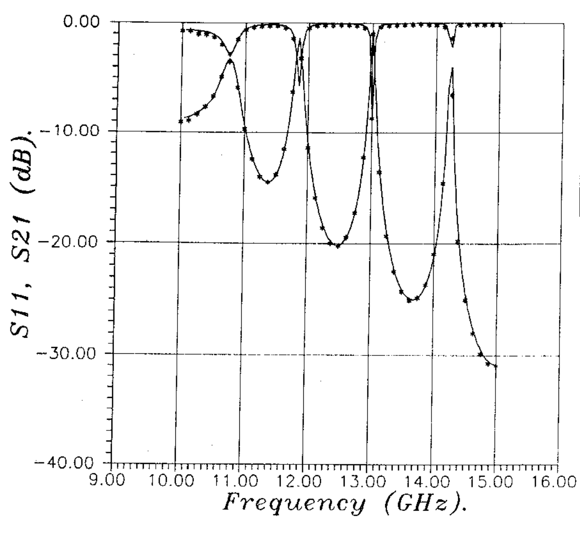


Fig. 6 Measured results of the low-pass filter designed by COM DEV Canada.



Input Waveguide	Width Height	19.05 mm 9.52 mm
Input Window Coupling	Width Height Thickness	14.00 mm 1.00 mm 0.50 mm
Circular Resonator	Radius Length	11.70 mm 100.00 mm
Output Window Coupling	Width Height Thickness	18.00 mm 1.00 mm 0.50 mm
Output Waveguide	Width Height	19.05 mm 9.52 mm

Fig. 7 Comparison between simulated results and measurements for a single circular resonator with different input and output rectangular couplings. The stars are the measured results while the full line represents the results obtained with our approach.